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Performance Prediction of Negative Temperature Coefficient Thermistor

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Abstract:
This paper presents the analysis of some aspects of gain linearization of a negative temperature co-efficient (NTC) thermistor using the mathematical tool based on partial differentiation and Taylor's series expansion. A suitable hardware scheme for the proposed gain linearization of the selected thermistor is also presented in this paper. Though the proposed analysis gives a closed form formulation for the gain linearization, to have a better feeling on the results involving parameters variations, the results were obtained by using suitable computer program. These simulation results were compared with the experimental results and found to agree to a reasonable extent.

1. Introduction:

Negative temperature co-efficient thermistors are presently used for practical applications in different protective and control systems. Due to its non-linear variation of resistance with respect to temperature, the NTC thermistor are most suitable for protective systems. The NTC thermistor are suitably used for control elements and as a control circuit consisting of OP-AMP or similar devices, the output/input characteristics of that control circuit also behaves like a non-linear. Therefore, the non-linearity of the control system required a appropriate gain linearization technique. Many researchers developed gain linearization improved technique of thermistor characteristics. In 1974 [Broughton (1)], developed six suitable circuits using the single thermistor for system response linearization. Out of the six circuits, there are three op-amp bridges, where the thermistor is placed in the input branch of the bridge. Such type of approach developed by Broughton has limitation as the condition for linearity obtained is mainly dependent on the specifications of the thermistor utilised in the particular circuit. Khan & Sengupta [2], have developed technique for linearization of output frequency of thermistor- astable multi-vibrator in 1981. The astable multi-vibrator incorporating this technique was applicable to offer an approximately linear temperature to frequency conversion in the range of 0^0 C to 800^0 C. This technique verified successfully in low temperature range from -500^0 C to 600^0 C.

Further, in 1984, again they developed a thermistor based temperature to frequency converter [3] using a delay network. This has been for gain linearization (i.e. Input/Output) relation over the range from 252^0 K to 528^0 K. In 1993, Kaligavardan et. al. [4], proposed new four curves fit for the resistance temperature characteristic of a thermistor. It has been proved better fit than two constant law $R=Ae^{B/T}$ and was demonstrated by taking the examples of two standard commercial thermistors. They have suggested a simple numerical methods for determining the constants of new fits and also develop a temperature to frequency converters employing the proposed fit. The scheme has been successful over a temperature range from 300 to 390^0 K.

Analysis and study on nonlinearity, in addition to gain linearization of thermistor, is also of importance in the present paper analyses sum aspects of gain linearization of a NTC thermistor using mathematical tools based on the partial differentiation and Taylor's series expansion. The paper also present s a suitable hardware substitute with possible results and discussions. The basis of the analysis is founded on a particular topology of the hardware circuit consisting of the particular NTC thermistor as a control element.

2. PROBLEM FORMULATION:

The resistance of a thermistor is a non-linear function of a temperature. This function can be expanded using a Taylor's Power series of the concerned independent variable. Such a series may theoretically be helpful but is practically unsuitable to predict the output of the concerned hardware scheme. Generally for mathematical analysis and for the purpose of design, such long series is truncated as per the discretion of the designer. Consequently, the authors of this paper also linearize the resulting power series as far the mathematical formulation is concerned.

Any mathematical analysis based on a suitable network topology. Such topology of a single thermistor-active bridge for the actual experimental circuit is shown in Fig. 1

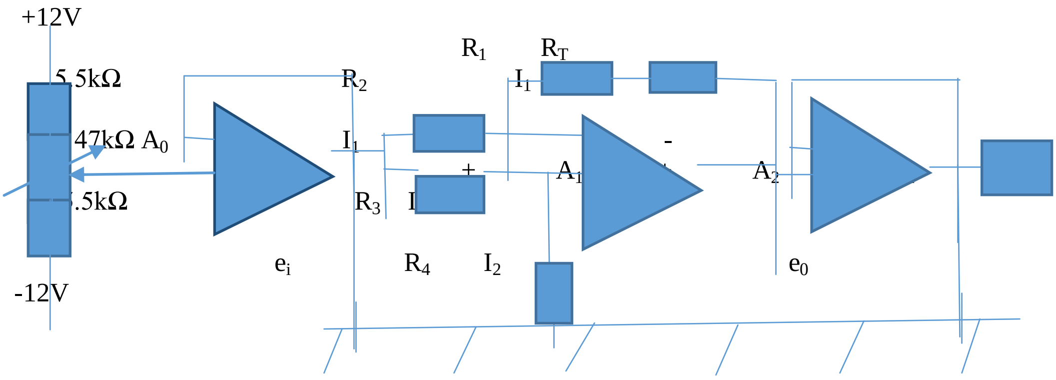

Fig. 1: Experimental Circuit Diagram of Single Thermistor Active Bridge

Following the topology of the experimental circuit shown in Fig. 1, the problem is formulated as presented below and the list of symbols is separately presented in the nomenclature. The resistance-temperature relationship for thermistor is expressed as given by

$$R_T = R_{T0}\, e^{[b\{(1/T) - (1/T_0)\}]} \qquad (1)$$

Let $R_1 + R_T = R'_1$ (2)

From Fig. 1, we have

$$V_2/e_i = R4/(R_4+R_3) \qquad (3)$$

Where V_2 is the potential difference of inverting and non-inverting terminals of operational amplifier A_1 and hence

$$(e_i - V_2)\, R'_1 = (V_2 - e_0)\, R_2 \qquad (4)$$

Equations (3) and (4), yields the gain equation

$$e_0/e_i = [R_4R_2 - (R_1+R_T)R_3]/[R_2\,(R_3+R_4)] \qquad (5)$$

Now dividing numerator and denominator of above equation by R_T R_4, we get gain equation as

$$e_0/e_i = [(R2/R_T) - \{(1+R_2R_1/R_2\,R_T)R_3/R_4\}/\{R_2(1+R_3/R_4)/R_T\}] \qquad (6)$$

Defining the ratios in equation (6) as below

$R_T/R_2 = r(x)$; $R_1/R_2 = \delta$; $R_4/R_3 = \alpha$; $b/T_0 = \beta$; $T/T_0 = x$; $R_{To}/R_2 = r_0$;

Also, defining the term e_o/e_i as k(x), then we can write equation (6) as below

$$e_0/e_i = k(x) = ((\alpha-\delta)-r)/(\alpha+1) \qquad (7)$$

After linear fraction transformation in r(x), K can be written as below

$$K = (a_0+a_1r)/(b_0+b_1r) \qquad (8)$$

Comparing equations (7) and (8), we get

$a_0 = \alpha-\delta$, $a_1 = -1$, $b_0 = \alpha$, $b_1 = 1/r$

ndard normalized form of equation can be written as below,

$$r(x) = r_0\, e^{(\beta(1/x) - 1)} \qquad (9)$$

For gain linearization, gain k(x) expressed in terms of Taylor's series expansion about x=1 and following expression is obtained,

$$k(x) = K(1)\,(x-1)^0 + [\delta k/\delta x]_{x=1}(x-1) + 1/2[\delta^2k/\delta x^2]_{x=1}(x-1)^2 + 1/6\,[\delta^3k/\delta x^3]_{x=1}\,(x-1)^3 + \ldots \qquad (10)$$

Equation (10) can be expressed in terms of constants up to third order differential terms as under

$K_0 = K(1)$; $K_1 = [\delta k/\delta x]_{x=1}$; $K_2 = [\delta^2k/\delta x^2]_{x=1}$; $K_3 = [\delta^3k/\delta x^3]_{x=1}$

Differentiating equation (8) w.r.t. "r", it yields

$$\delta k/\delta_r = (a_1b_0 - a_0b_1)/(b_0 + rb_1)^2 \qquad (11)$$

Normalised equation of r(x) given by equation (9) is partially differentiated w. r. t.'x' to obtain

$$\delta r/\delta_x = -r\beta/x^2 \qquad (12)$$

Now from equations (9) and differentiating equations (11) and (12), Finally, we get the following relationships as below:

$$K_0 = (a_0+a_1r)/(b_0+b_1r) \qquad (13)$$

$$K_1 = [\delta k/\delta x]_{x=1} = -r\beta(a_1b_0 - a_0\,b_1)/(b_0+b_1r_0)^2 \qquad (14)$$

$$K_2 = [\delta^2k/\delta x^2]_{x=1} = [-r\beta(a_1b_0 - a_0\,b_1)/(b_0+b_1r_0)^3][1+\beta(b_0-b_1r)/2(b_0+b_1r)] \qquad (15)$$

$$\text{Or, } K_2 = -K_1\,(1+B) \qquad (16)$$

Where,

$$B = \beta(b_0-b_1r_0)/2(b_0+b_1r_0) \qquad (17)$$

Also,

$$K_3 = [\delta^3k/\delta x^3]_{x=1} = K_1\,(1+2B+C) \qquad (18)$$

Where,

$$C = \beta^2(b_0^2 - 4b_0b_1r_0 + b_1^2r_0^2)/6(b_0 - b_1r_0)^2 \qquad (19)$$

Now substituting the value of a_0, a_1 , b_0 , b_1 in equation (16) and (18), we obtain

$$B = \beta (\alpha-1)/2(\alpha+1) \quad (20)$$

And,

$$C = \beta^2(\alpha^2 -4\alpha+1)/6(\alpha +1)^2 \quad (21)$$

For gain linearization, second and all higher order co-efficient of the Taylor's series are made equal to zero in the operating temperature range.

Now,

$$K_2 = -K_1 (1+\beta)=0$$

$$\text{Or, } \beta= -1$$

Now from equation (20), we obtain

$$\alpha = (\beta-2)/(\beta+2) \quad (22)$$

and,

$$C = (-\beta^2+12)/12 \quad (23)$$

Again, since $K_0 = 0$,

Hence,

$$r_0 = \alpha-\delta \quad (24)$$

From equations (7), (9) and (23), we get

$$k(x) = [(\alpha-\delta)-\{(\alpha-\delta) e^{(\beta(1/x) - 1)}\}]/(\alpha+1) \quad (25)$$

Now,

$$K_3=K_1(-\beta^2/12) \quad (26)$$

Fractional deviation "D" from the linearity is defined by expression as below:

$$D = [k(x)-K_1(x)]/K_1(x)$$

Where,

$$k(x)=K_1(x-1)+ K_3(x-1)^3+\ldots\ldots\ldots\ldots \text{as } K_0 = K_2 = 0$$

Now we obtain,

$$D= [K_3(x-1)^3]/ K_1(x-1)$$

Or, $$D=K_3 (x-1)^2 /K_1$$

Or, $$D= [\beta^2 (x-1)^2]/12 \quad (27)$$

3. HARDWARE IMPLEMENTATION

The numerical result based on the above problem formulation shown in Fig. 3 can be verified with the experimental results. The experimental circuit in connection with the hardware scheme is shown in Fig.2 with some necessary modification of Fig. 1.

The experimental circuit shown in Fig.2 comprises of mainly some resistance and OP-AMPS to have a shape of active bridge circuit. The first part of the circuit produces a constant DC voltage which is obtained from the output of the OP-AMP A_0. At the input of the same OP-AMP, a potential divider is connected to inject a variable dc vo0ltage.

With reference to Fig.3, the scheme is basically a differential amplifier used to amplify the output from a transducer bridge consisting of the arm resistances ($R_1 + R_T$), $R2$, R_3 and R_4 respectively. Due to change in the temperature, whenever R_T will change from one value to another value, the null condition of the said transducer will be disturb. As a result operational amplifier output voltage "e_o" will be changed depending on the operating temperature.

In context to the present discussion on the basis of hardware scheme, it can be mentioned that resistances $R2$, R_3 act as the role of load as viewed from the input voltage source "e_i". These loads may be heavy and to compensate the equivalent loading effect, a voltage follower may be used as a buffer amplifier at the output of the concerned OP-AMP. In the present case the OP-AMP (A_3) is used as a unity gain buffer amplifier (Voltage follower).

The result of output voltage "e_o" is obtained from the experiment performed and these experimental results can be compared with the simulation results. The combined simulation and experimental results and their discussions are presents.

4. OBTAINED RESULTS

With reference to the problem formulation presented in section 1, equation (7) appears as the main mathematical expression which clearly indicates the objective of the present research work. With reference to the same equation, "r" is the function of temperature and other parameters taken as constant. The parameter K in equation (8) is basically a non-linear function of "x" where "x" represents the term "T/T_0" with T being the general temperature in degree K and T_0 being the original temperature (fixed) in degree K. With refrence to equation (16), K_0 by varying has been assumed to be zero and on this basis, the combinations of equations (7),

(8) and (29), ultimately give a compact mathematical algorithm which gives the various values of the voltage, e_0 obtained at the output of the bridge circuit shown in Fig.2 with variation imposed on the parameter δ. This algorithm is presented in Fig. 3, stage-wise, starting from fundamentals. Based on the above said discussions and algorithm, simulation results are presented in tables 1, 2, 3 and 4. Each of these mathematical tables represents the calculated values of r(x), e_0, k(x) as a function of operating temperature (T) for a fixed value of δ. It is a matter of interest to note that simultaneous changes in operating temperature (T) and the value δ cannot be experimentally implemented because in that case a change in δ (e. i.) and equivalent change in R_1 would cause a change in feedback path current of the operational amplifier "A_1" shown in Fig. 1 along with the change in R_T due to change in temperature. Hence in such a case, confusion have been created about the fact whether the output voltage "e_0" is affected by δ or T. To avoid such confusion expected to be appeared in the experimental stage, the calculations have been carried out by varying T in the simulation stage for a fixed value of δ. The fixed value of δ has been taken as 0, 0.1, 0.2 and 0.3 in tables 1, 2, 3 and 4 respectively. With reference to the hardware scheme shown in Fig. 3, by varying the temperature T the output voltage, "e_0" has been experimentally measured by keeping δ as fixed. These results are presented in tables 5, 6, 7 and 8 respectively. The values of δ have been mentioned in tables 1-4 indicating the simulation results. The comparison of the experimental results with simulation results, for a fixed value of δ, clearly shows a reasonably good agreement between the results. For having a better feeling on the performance of the experimental scheme, by varying the temperature T, gain value of the OP-AMP system have been presented in tables 9, 10, 11 and 12 respectively for fixed value of δ taken earlier i. e. δ=0, 0.1, 0.2 and 0.3 respectively.

4. RESULTS AND DISCUSSIONS

Though there is a reasonably good agreement between the calculated and experimental values of "e_0" as observed in tables 1-4 1nd tables 5-8, still it is needed to indicate the percentage error of experimental with respect to simulation results. Such necessity arises due to the purpose of designing a better hardware scheme based on the existing scheme. In other words, the percentage errors in the experimental results may be treated as a design tool by an instrument designer. The percentage errors in the experimental results are presented in tables 9-12 for fixed value of δ i. e. 0, 0.1, 0.2 and 0.3 respectively.

An increase in δ corresponds to an increase in R_1 for a fixed value of R_2. Equation (5) clearly shows that e_0 will decrease due to increase in R_1 at a fixed value of e_i and other parameters. But in the experimental stage, e_0 will further decrease as compared to calculated values because R_1 will further increase due to effect of temperature. This effect is a natural phenomenon because R_1 is in proximity of R_T. As the experimental value of e_0 is lower than its calculated value, percentage error increases. The effect of temperature on R_1 cannot be sensed from the equation formulated because the theoretical modelling of R_1 as a function of temperature has not been developed.

Table 1		Table 2		Table 3		Table 4	
Temperature	e_0	Temperature	e_0	Temperature	e_0	Temperature	e_0
298	0.000	298	0.000	298	0.000	298	0.000
306	0.100	306	0.090	306	0.076	306	0.058
314	0.178	324	0.152	314	0.125	314	0.100
322	0.228	322	0.200	322	0.162	322	0.130
330	0.265	330	0.230	330	0.193	330	0.153
338	0.302	338	0.258	338	0.215	338	0.170
346	0.325	346	0.280	346	0.231	346	0.182
354	0.345	354	0.294	354	0.244	354	0.193
362	0.358	362	0.306	362	0.258	362	0.203
368	0.370	368	0.316	368	0.262	368	0.210

Table 5		Table 6		Table 7		Table 8	
Temperature	K(x)	Temperature	K(x)	Temperature	K(x)	Temperature	K(x)
298	0.000	298	0.000	298	0.000	298	0.000
306	0.094	306	0.085	306	0.072	306	0.055
314	0.168	324	0.143	314	0.118	314	0.094
322	0.215	322	0.2189	322	0.153	322	0.123
330	0.250	330	0.217	330	0.182	330	0.126
338	0.258	338	0.243	338	0.203	338	0.140
346	0.307	346	0.264	346	0.218	346	0.160
354	0.325	354	0.277	354	0.230	354	0.172
362	0.3338	362	0.289	362	0.243	362	0.191

368	0.349	368	0.292	368	0.247	368	0.198

Table 9		Table 10		Table 11		Table 12	
Temperature	% Error	Temperature	%Error	Temperature	% Error	Temperature	% Error
298	0.000	298	0.000	298	0.000	298	0.000
306	2.000	306	7.778	306	8.210	306	6.897
314	4.495	324	5.263	314	4.800	314	6.000
322	2.193	322	5.000	322	3.704	322	5.385
330	0.377	330	2.609	330	4.145	330	5.229
338	2.649	338	3.101	338	4.186	338	4.706
346	2.154	346	3.214	346	3.614	346	3.846
354	2.319	354	2.721	354	2.279	354	3.627
362	1.676	362	2.288	362	4.651	362	4.433
368	2.432	368	0.968	368	3.435	368	5.238

5. CONCLUSIONS

The papers highlights the following conclusions:

- The mathematical formulation for the system operating parameters are most useful for gain linearization.
- The gain linearization technique using truncation of Taylor's Series gives sufficient accuracy in the results.
- Errors between the experimental and the simulation results of the output voltage plays an important role for designing improved hardware scheme.
- A proper theoretical modeling of R_1 as a function of temperature can explain an increase in percentage error of experimental value of output voltage as compared to its calculated value.

REFERENCES

[1] Broughton, B. (1974) "Analysis and design of almost linear one thermistor temperature transducer.

"IEEE Trans. Inst. And Meas., 23(1).

[2] Khan, A and Sengupta, R.S. (1981) "A linear frequency to temperature converter using a thermistor."IEEE Trans. Inst. Meas., 30(4).

[3] Khan, A and Senggupta, R. S. (1981) "A linear thermistor based temperature to frequency converter using network." IEEE Trans. Inst. Meas., 34(1).

[4] Morty, V. G. K., Sankaran, S. and Kliyugavardan, S. (1993) "A new compensation for thermistor and its implementation for response linearization of over a wide temperature range." IEEE Trans. Inst. Meas., 42(5).

[5] Ibenez-Diaz, J., Pantaleon, C., Santamarya, I. Fernandez, T. and Martinez, D. (2001) "Non-linearity-estimation in power amplifier based on sub-sampled temporal data." IEEE Trans. Inst. Meas., 50(4).

[6] Sengupta, R. S. and Sankaran, S. (2007) " Mathematical modeling of thermistor based temperature for gain linearization." IEEE Trans. Meas. 412(5).